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ON EINSTEIN-RANDER'S METRIC

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ABSTRACT

We study a characteristic condition of Einstein-Rander's metrics, we prove that a non-Riemannian Rander's metric $F = \alpha + \beta$ is Einstein metric. By using the data (h, W), it is proved that an n-dimensional (n≥2) Rander's metric $F = \alpha + \beta$ is having projective changes between a Finsler space with(α, β)-metric and the associated Riemannian metric.

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KEYWORDS: *Finsler Space, Rander's Metric, Navigation Data*

1. INTRODUCTION

In this paper, we study F^n be n-dimensional Finsler space equipped with metric function $L(x, y)$. In the geometry of Finsler spaces, let F be a Finsler metric. F is called as Einstien scalar σ if

$Ric = \sigma F^2$ 1.1

where $\sigma = \sigma(x)$ is a scalar function on M. F is said to be Ricci constant if F satisfies the above condition

where $\sigma = const.$

Recently some results have been drawn on Finsler-Einstein metrics of (α, β) type. The (α, β)-metrics form aclass of Finsler metrics appearing in Physics, Biology, Control Theory, etc. D. Bao and C. Robles derived Einstein Randers metric of dimension $n(\geq 3)$. A3-dimensional Randers metric is Einstein if and only if it is of constant flag curvature.For every non-Randers (α, β)-metric $F = \alpha\varphi(s), s = \alpha/\beta$.

In this paper it is investigated Einstein Rander's metrics $F = \alpha + \beta$, for which were stricted the consideration to the domain where $\beta = b_i(x)y^i > 0$. By using a computation, we obtain the characteristic conditions of Einstein Rander's metrics in Theorem 1.1, which generalize the result.

An (α, β)-metric, if $r_{ij} = 0$ the metric is called Killing form. $\beta$is said to be a constant Killing form if it is a Killing form and it satisfies the condition$r_{ij} = 0, s_i = 0$.

For (α, β)-metrics with constant Killing form, Einstein Kropina metrics, we have the following theorem.

Theorem 1.1: Let $F = \alpha + \beta$ be a Rander's metric with Killing form β on an n-dimensional manifold M, $n \geq 2$. In this case, $\sigma = \frac{1}{4}\lambda b^2 \geq 0$, where $\lambda = \lambda(x)$ is the Einstein scalar of α. F is Ricci constant when n ≥ 3.

Rander's Metric using Ricci Curvature

If F is a Finsler metric on an n-dimensional manifold, defined by

$.G^i := \frac{1}{4} g^{il}\{[F^2]_{x^k y^l} y^k - [F^2]_{x^l}\}$

For any $x \in M$, $y \in T_x M\{0\}$, the Riemann curvature $R_y := R^i_k \frac{\partial}{\partial x^i} \otimes dx^k$ is defined by $R^i_k := 2\frac{\partial G^i}{\partial x^l} - \frac{\partial^2 G^i}{\partial x^n \partial y^l} y^n + 2G^n \frac{\partial^2 G^i}{\partial y^n \partial y^l} - \frac{\partial G^i}{\partial y^n}\frac{\partial G^n}{\partial y^l}$

$Ric := R^n_n$. By definition, an (α, β)-metric on M *is* in the form $= \alpha\phi(s), s = \frac{\beta}{\alpha}$, where$\alpha = \sqrt{a_{ij}(x)y^i y^j}$ is a Riemannian metric, $\beta = b_i(x)y^i$ a 1-form. It is known that (α, β)-metric with$\left\|\beta_x\right\|_\alpha < b_0$is a Finsler metric if and only if $\phi = \phi(s)$ is a positive smoothfunction in an open interval $(-b_0, b_0)$ satisfies the following condition: $\phi(s) - s\phi'(s) + (b2 - s2)\phi''(s) > 0, \forall|s| \le b < b_0$, see [7].Let$r_{ij} = \frac{1}{2}(b_{k|l} + b_{l|k}), sij = \frac{1}{2}(b_{k|l} + b_{l|k})$, where "$|$" denotes the covariant derivative with respect to the Levi-Civita connection of α.Denote$r^i_j := a^{ik} r_{kj}, r_j := b^i r_{ij}, r := r_{ij} b^i b^j = b^j r_j, s^i_j := a^{ik} s_{kj}, s_j := b^i s_{ij}$, where$(a^{ij}) := (a_{ij})^{-1}$and $b^i := a^{ij} b_j$. Denote $r^i := a^{ij} r_j, s^i := a^{ij} s_j$, $r_{i0} := r_{ij} y^j$, $s_{i0} := s_{ij} y^j$, $r_{00} := r_{ij} y^i y^j$, $r_0 := r_i y^i$ and $s_0 := s_i y^i$.If G^i is the geodesic coefficient of F and $\bar{G}^i$ is the geodesic coefficients of α. Then we prove the following lemma.

Lemma 1.1:

For an (α, β)-metric $= \alpha\phi(s), s = \frac{\beta}{\alpha}$, the geodesic coefficients G^i are given by$G^i = \bar{G}^i + \alpha Q s^i_0 + \psi(r_{00} - 2\alpha Q s_0) b^i + \frac{1}{\alpha}\Theta(r_{00} - 2\alpha Q s_0) y^i$ (1.2)where $Q := \frac{\phi'}{\phi - s\phi'} = 1, \psi := \frac{\phi''}{2[\phi - s\phi' + (b^2 - s^2)\phi'']} = 0,$

$$\Theta := \frac{\phi\phi' - s(\phi\phi'' + \phi'\phi')}{2\phi[\phi - s\phi' + (b^2 - s^2)\phi'']} = \frac{1}{2(1+s)}$$

We consider a special (α, β)-metrics which is called Rander's-metric with the form$F = \alpha\phi(s)$, $\phi(s) := s^{-1}$, $s = \frac{\alpha}{\beta}$

We get the Ricci curvature of Rander's metric by using Lemma 1.1.

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